Ethnomathematics: Ideofacts, Sociofacts, and Artefacts of Mathematical Context in Yogyakarta

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ABSTRACT

Aim. The purpose of this study is to explore ideofacts, sociofacts, and artifacts that have the potential to serve as contexts for teaching mathematics in Yogyakarta.

Methods. This qualitative research employs an ethnographic approach. The study investigates potential locations including the Kotagedhe Mataram Mosque, Prambanan Temple, and the Planting Plantation System in Yogyakarta. Instruments used include interview guides, observation sheets, and artifact exploration Tables. Key informants were selected at each research site to provide ethical insights from an emic perspective, resulting in dialectical data. To ensure data validity, the researchers sutilised data triangulation (interviews, observations, and documentation) and conducted focus group discussions (FGDs) with cultural experts and ethnomathematicians to review and discuss the findings.

Results. The findings indicate that each location contains mathematical content applicable to teaching, particularly in geometry (both two-dimensional and three-dimensional), pre-algebra (multiplication), and number theory (least common multiple).

Conclusion. The study demonstrates how cultural contexts, such as the Kotagedhe Mataram Mosque and Prambanan Temple, can be integrated into elementary mathematics education, illustrating mathematical concepts such as spatial geometry, symmetry, and multiplication. It identifies potential learning tools, including folded paper, flat-shaped paper, LEGO, and Augmented Reality, to enhance the teaching of these concepts. Additionally, a Hypothetical Learning Trajectory (HLT) is developed to guide the learning process based on these cultural contexts and tools.

Keywords: artefact, ethnomatematics, ideofact, mathematical contexts, sociofact

Introduction

The use of context in mathematics education is crucial. According to Marja van den Heuvel-Panhuizen and Paul Drijvers (2014), there are three main benefits of using context: (a) facilitating access to problems, (b) making problems more visible and elastic, and (c) assisting in finding a solution strategy. Experts suggest that using context makes mathematics more meaningful and easier for students to understand; however, selecting an appropriate context for studying mathematics is complex and multidimensional. Certain contexts can guide students in developing and rediscovering mathematical tools and concepts (van den Heuvel-Panhuizen, 2020).

Despite these benefits, students often struggle with mathematics in a socio-cultural context. Research by O. Arda Cimen (2014); and Desinta Yosopranata et al. (2018) indicates that students have difficulty solving mathematical problems within a cultural context. One reason for this difficulty is their challenge in understanding contextual sentences, which relates to their multi-representational abilities, especially in textual representations (Garderen & Montague, 2003). Sukoriyanto Sukoriyanto et al. (2016) also found that Indonesian students' problem-solving abilities are lacking, primarily because of their difficulties in understanding the context of problem-solving questions (Deliyianni et al., 2016). This inability to understand contextual problems ultimately affects their performance on assessments like *Asesmen Kompetensi Minimum* (AKM) and Programme for International Student Assessment (PISA).

Teachers also face challenges in integrating cultural context into mathematics instruction. Interviews with five teachers in various Yogyakarta schools revealed that they struggle to incorporate cultural contexts when teaching mathematical concepts. This finding is supported by research from Anna-Maija Partanen (2011); and Uba Umbara et al. (2021), which states that teachers often find it difficult to connect socio-cultural context with mathematical content in the classroom. This is ironic given that Yogyakarta is renowned for its rich cultural heritage. Teachers should have many cultural contexts available to enhance mathematics learning.

Ethnomathematics addresses this issue by repositioning mathematics, often seen as rigid, to be more democratic and inclusive of different cultural perspectives (Faiziyah et al., 2020; Prahmana et al., 2021). Ethnomathematics is both a pedagogical and didactic innovation that bridges cultural context and mathematical content (Prahmana, 2021).

Several studies have developed thematic-based learning tools. For example, Lalu Muhammad Fauzi et al. (2022) created mathematics learning media based on Sasak cultural architecture, using traditional Sasak houses as a context for spatial learning. Similarly, Susiana et al. (2020) developed ethnomathematics-based worksheets focused on Batik Tapis Lampung to enhance students' geometric transformation abilities in both two-dimensional and three-dimensional spaces. However, these efforts have predominantly focused on geometric content, with less development in numerical contexts such as sequences, modulo, or number patterns (Maulyda et al., 2023).

This research aims to explore ideofacts, sociofacts, and cultural artifacts in Yogyakarta to identify richer cultural contexts that can be integrated with a broader range of mathematical content, beyond just geometry. The findings will provide a variety of cultural contexts for teachers to use in mathematics instruction, enhancing the educational experience and making it more relevant and engaging for students.

Method

Research Design

This type of qualitative research uses an ethnographic approach to explore ideofacts, sociofacts, and artifacts in Yogyakarta that can potentially become a mathematical context (Creswell, 2014). The ethnographic approach is used because it can explore socio-cultural phenomena in Yogyakarta. Researchers use secondhand experiences where researchers will be involved as observers who interpret data from outside (emic). However, researchers still use key persons as the main source of information (ethics). Thus, the ethnographic point of view used in this study is a combination of ethical and emic viewpoints, or dialectic (Geertz, 1973).

Measuring Ideofacts, Sociofacts, and Artifacts

Measuring Ideofacts

Ideofacts in this study refer to intangible cultural elements such as beliefs, values, norms, and philosophies embedded within the communities at the research locations. These ideofacts serve as the foundational layer of culture, shaping the way individuals perceive and relate to mathematical ideas in their socio-cultural context. To measure ideofacts, semi-structured interviews were conducted with key informants at each research site. The questions were designed to uncover the mathematical relevance of local narratives, beliefs, and symbolic meanings. For instance, questions at the Kotagedhe Mataram Mosque focused on geometric patterns and numeric symbolism in religious rituals, while at Prambanan Temple, the inquiry centred on Hindu cosmological concepts reflected in temple architecture. At the Kedaulatan Rakyat Newspaper, the investigation explored how editorial decisions involve implicit mathematical reasoning. By analysing these narratives, the study captures the deep cultural connections between ideofacts and mathematical contexts. The indicators of the instruments show in Table 1.

Table 1
Ideofact Indicators Measurement

Indicator	Questions
Beliefs related to mathematics	What are the cultural values or beliefs that reflect mathematical thinking?
	How do local traditions incorporate mathematical concepts (mathematical content)?
Numerical perception in traditions	How do people in this community perceive numbers in daily or ritual activities?
	Are there any specific symbols or patterns that represent numerical ideas in local traditions?
Cultural narratives	Can you share any historical stories or myths that include numbers or calculations?
	How are mathematical ideas embedded in traditional cultural narratives?

Source. Elaboration by Partanen (2011) and Geertz (1973).

Measuring Sociofacts

Sociofacts pertain to the social systems, interactions, and relationships within the communities observed at the research sites. These elements highlight the communal practices and organisational systems that indirectly or directly incorporate mathematical thinking. Data collection for sociofacts relied on observations recorded using an Observation Sheet. Specific aspects included the examination of daily rituals, ceremonies, and organisational practices. For instance, at the Kotagedhe Mataram Mosque, observations were made on how prayer schedules were calculated and how communal funds were managed. At Prambanan Temple, the focus was on the rituals involving offerings and their potential numerical or proportional bases. Similarly, at the Kedaulatan Rakyat Newspaper, the analysis explored the coordination of printing processes and numerical data handling in journalistic practices. These observations provide insights into how socio-cultural systems incorporate and reflect mathematical principles in their daily operations. The indicators of the instruments show in Table 2.

Table 2
Sociofact Indicators Measurement

Indicator	Questions
Mathematical patterns in activities	What activities or rituals involve mathematical patterns or sequences (mathematical content)?
	How do these patterns contribute to the cultural or social identity of the community?
Social role and hierarchy	How are mathematical concepts used in organising social roles or resource distribution?
	Are there specific practices that highlight mathematical reasoning in social systems?
Cultural systems	What are the main cultural practices that influence the use of mathematics in daily life?
	How does the social system encourage or depend on mathematical thinking?

Source. Elaboration by Partanen (2011) and Geertz (1973).

Measuring Artifacts

Artifacts refer to tangible cultural objects and physical evidence present at the research locations, such as architectural structures, preserved historical items, and tools. These artifacts often embody mathematical principles such as geometry, symmetry, measurement, and proportion. The measurement of artifacts was conducted using an Artifact List instrument, which included a detailed inventory of physical items and their descriptions. At the Kotagedhe Mataram Mosque, the focus was on documenting the dimensions and designs of architectural elements, such as the main prayer hall and decorative patterns. At Prambanan Temple, measurements of temple structures, staircases, and relief panels were analysed to identify geometric and symmetrical properties. At the Kedaulatan Rakyat Newspaper, tools and templates used in layout design were examined, as well as archived records showcasing mathematical relevance. These tangible items provide concrete evidence of the integration of mathematical concepts within cultural artifacts at the research sites. The indicators of the instruments show in Table 3.

Table 3
Artifact Indicators Measurement

Indicator	Questions
Geometric structure of buildings	What geometric shapes or patterns are visible in the design of this building?
	Are there specific proportions or measurements used in the construction?
Patterns in decorative objects	What repetitive patterns or designs are present in the decorations or carvings?
	How do these patterns reflect numerical or mathematical ideas?
Historical documents or tools	What tools or documents are preserved, and how are they used in mathematical contexts?
	Are there historical records that show evidence of mathematical concepts?

Source. Elaboration by Partanen (2011) and Geertz (1973).

Research Locations and Informants

In this study, there are three prominent locations to be explored. The three locations are (a) Kotagedhe Mataram Mosque; (b) Prambanan Temple; and (c) Kedaulatan Rakyat Newspaper (KR). Each research location will select one key person as the primary informant. For the Kotagedhe Mataram Mosque site, the primary source is Warsiman as head of security at the Kotagedhe Mataram Mosque. For Prambanan Temple, two speakers, Chandra, and Rosita, were chosen as conservation team members at the DI Yogyakarta Cultural Heritage Preservation Centre. Meanwhile, from the KR Newspaper, one source will be selected, whose name is not mentioned due to personal reasons, as one of the curators at the newspaper.

Data Collection and Procedures

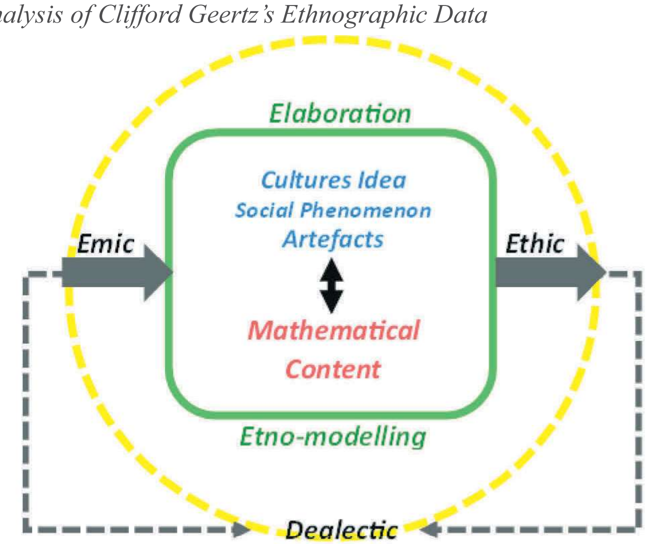
The process of collecting data in this study uses three main instruments, namely, (a) Interview Guidelines; (b) Observation Sheet; and (c) List of Artifacts. The Interview Guide consists of 8 semi-structured questions that can be developed during the interview process. This instrument is used to explore video fact information that has the potential to have a mathematical context. Interviews were conducted with key informants while observing the research location.

The Observation Sheet instrument consists of 3 main observational aspects: the social system, cultural system, and mathematical content. This instrument is used to observe cultural activities in the research location. This instrument is used to explore socio-fact information that has the potential to have a mathematical context. Observations were carried out comprehensively, accompanied by guides in the research locations. Finally, the artifact list instrument was used to collect artifact data at each research location.

Data Analysis

Ideofak, sociofacts, and artefacts data that have been collected were then analyse using the ethnographic analysis of Clifford Geertz (1973). There are several stages in the ethnographic data analysis process, among others; (a) make connections between data findings (patterns, relationships, similarities, or points of contrast); (b) make notes regarding the interrelationships between the data obtained; (c) Reduction of unrelated or irrelevant data to deepen. In this study, the data analysis process, especially coding, used the NVivo software. The data analysis procedure can be seen in Figure 1 below.

Figure 1
Analysis of Clifford Geertz's Ethnographic Data



Source. Own elaboration.

Result and Discussion

Kotagedhe Mataram Mosque

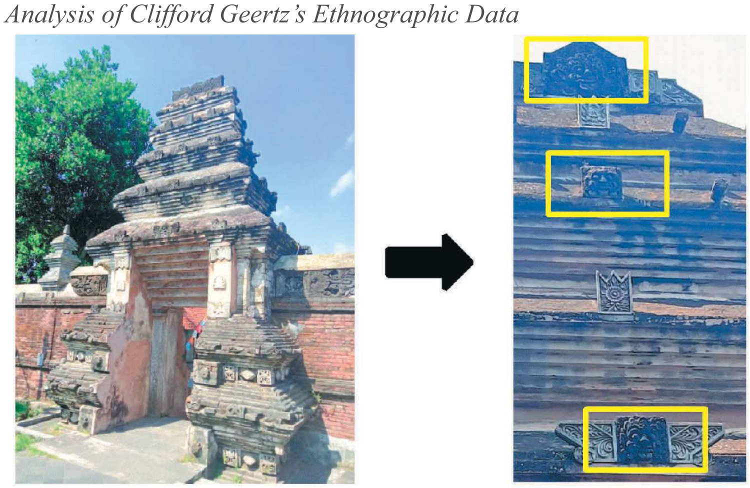
The first potential location to be searched is the Kotagedhe Mataram Mosque. To obtain valid information, the researcher chose a key person named Mr Warsiman, who currently serves as the chief guard of the Kotagedhe Mataram Mosque. The results of the interviews, which were conducted for approximately 1.5 hours, the researchers obtained some cultural information. This cultural information is an emic categorisation from crucial persons in potential locations.

The Kotagedhe Mosque was founded in 1586 and was initiated by Panembahan Senapati, who Sunan Kalijaga directly ruled as a teacher. One of the main objectives of building the Kotagedhe Mosque is to spread Islam in the hinterlands (because the coastal areas have already been developed). In disseminating this Islamic religion, four things must be built and developed, namely, (a) Mosque➔Center of Worship; (b) Market➔Economic Center; (c) Alun-Alun➔Socializing Center; and (d) Keraton➔Center of Government. The Kotagedhe Mosque is a worship centre built at that time to be a place to give da'wah and perform community worship around Mataram.

Other information from the critical person relates to the ornaments around the Kotagedhe Mosque. The first ornament is the gate of the mosque which resembles a temple. The philosophical foundation of the shape of the mosque's entrance, which resembles a temple, is the assistance of Hindu residents in constructing the gate. This is due to using teak wood from Blora as the primary material for creating the mosque. The shape of the Kotagedhe Mataram Mosque gate can be seen in Figure 2 below.

Figure 2

Analysis of Clifford Geertz's Ethnographic Data

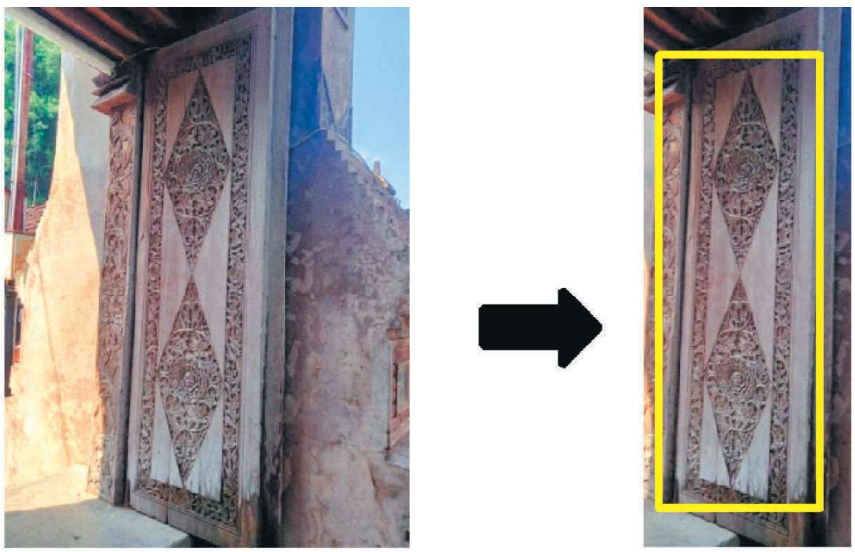


Source. Author's documentation.

In this gate, there are several carvings called Kalamangkara or a combination of the words Kala and Angkara which means rejecting reinforcements or rejecting bad things. This philosophy belongs to Hindus, but it was acculturated by Panembahan Senapati to be implemented at the Kotagedhe Mataram Mosque. Apart from that, the next ornament is the gate of the Kotagedhe Mataram Mosque which can be seen in Figure 3 below.

Figure 3

Gate of Kotagedhe Mataram Mosque



Source. Author's documentation.

Figure 3 shows the carved shape of the Kotagedhe Mataram Mosque gate, which resembles a diamond shape. Wajik is a typical Javanese food that is shaped like a rhombus. The choice of this diamond food object comes from a verse of the Koran in Surah Al-Fajr verse 23. "*Wajii-a yaumaidzin bijahannam, yaumaidzin yatadzakkarul insaanu wa anna lahudz dzikraa*". Meaning, "On that day, the devil will come. On that day, humans will remember humans for all their actions, but what's the point of remembering when it's too late?". This sentence reminded Muslims to always use the Kotagedhe Mataram Mosque as a place to worship.

Furthermore, the researcher interpreted the cultural information obtained to obtain the mathematical content contained in the cultural context of the Kotagedhe Mataram Mosque (interpretation with an ethical approach). The search results on the ornaments of the Kotagedhe Mataram Mosque yielded two mathematical content that could potentially be used in learning mathematics in elementary schools. The first ornament is the gate of the Kotagedhe Mataram Mosque in the form of a temple. The mathematical content contained in this temple artifact is Spatial Geometry. According to Iva Vojkuvkova (2012), spatial geometry ability is a person's skill in understanding and constructing geometric shapes. The representation of the geometric arrangement of the temple buildings can be seen in Figure 4 below.

Figure 4

Representation of the Geometrical Arrangement of the Gates of the Kotagedhe Mataram Mosque

Source. Author's documentation.

The representation of a temple-shaped flat shape can be associated with various geometrical concepts such as area, circumference, and height. Arranging shapes to resemble the shape of natural objects can also improve children's spatial geometry abilities; Lina Anggita Ahsani et al. (2022) research results state that children's spatial geometry abilities can be enhanced by arranging objects. This can improve children's sense of plane and space. This research is also supported by Gabriela Pavlovičová et al. (2022), which states that spatial geometry abilities can be identified from three activities, namely, (a) Arranging shapes based on shape; (b) Arranging shape based on size; and (c) Arrange shapes based on weight.

Potential learning tools that can be used are cube, beam, trapezoidal, and pyramid-shaped teaching aids. The child then arranges these props to resemble the original shape to calculate the area, circumference, and height. Support can be made in various sizes so children can be creative in arranging shapes based on their profile, size, and weight. If there is an Augmented Reality (AR) technology facility, researchers can use it as a reference object for observation for children. The description of the *hypothetical learning trajectory* (HLT) for this learning can be seen in Table 4 below.

Table 4
HLT of Flat Shape Spatial Geometry Capability

Hypothetical learning trajectory	Learning activity
L1+L2+L3	Measure the area and perimeter of the overall shape.
Triangle Area = ½ x a x t Triangle Circumference = s + s + s Rectangle Area = p x l Rectangle Circumference = 2 x (p + l) Trapezoid Area = ½ × t (a + b) Trapezoid Circumference = s + s + s + s	Take measurements of the area and circumference of each flat object that is arranged.
	Arranging flat shapes resembling the gate of the Kotagedhe Mataram Mosque which is shaped like a temple according to the height and shape of the object.
	Identify flat shapes scanned from Augmented Reality projections that can be used as materials for preparing temple buildings.
	Observing the object of the Kotagedhe Mataram Mosque Gate which is shaped like a temple.

Source. Research elaboration and Author documentation

The second ornament containing mathematical content is the door carving at the Kotagedhe Mataram Mosque. The mathematical content contained in this mosque door carving is folding symmetry. Folding symmetry material in elementary schools has been taught to ensure students have sufficient geometry skills to enter geometrical material (pre-requisite material) (de Walle et al., 2016). The representation of the folding symmetry that appears at the door of the Kotagedhe Mataram Mosque can be seen in Figure 5 below.

Figure 5

Representation of the Folding Symmetry on the Door of the Kotagedhe Mataram Mosque

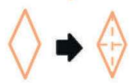
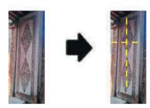
Source. Research documentation.

The concept of folding symmetry can be learned by students as a prerequisite material for studying geometric shapes. According to Stephen Whitelam et al. (2015), symmetry in elementary schools includes several sub-concepts of geometry: translation, rotation, reflection, and tessellation. The study's results, Asuman Duatepe-Paksu et al. (2012), stated that students in grades 3–5 should be able to apply transformations and use symmetries to analyse mathematical situations. In addition, students must also be able to predict and describe the results of sliding, flipping, and rotating two-dimensional shapes. Furthermore, according to Van Hiele (1973) in Wu-Yuin Hwang et al. (2023), the ability to fold symmetry is at the level of geometric analysis thinking (level 2). This ability allows students to analyse the field in terms of its components and their relationship with these components and look for empirical characteristics or rules for classifying the area.

The context found in the diamond carving on the door of the Kotagedhe Mataram Mosque has the potential to become a starting point for students to understand the concept of folding symmetry. Teachers can use rhombus-shaped paper as a manipulative medium that students can fold. Teachers can use AR technology to show students how symmetrical objects exist in the real world. In addition, with AR technology, teachers can also guide the process of folding things using videos visualised by AR. The description of the HLT for this learning can be seen in Table 5 below:

Table 5

HLT of the Fold Symmetry Concept

Hypothetical learning trajectory	Learning activity
Fold symmetry is the number of folds that exist in a flat or two-dimensional shape	Understanding the definition of folding symmetry in a plane (two dimensions).
	Understand the process of folding symmetry according to the previously drawn diagonal lines.
	Draw two rhombuses resembling the Carving of the Gate of the Mataram Kotagedhe Mosque which is shaped like a diamond.
	Draw objects on the paper provided. Students draw a rhombus along with its diagonal lines.
	Observing the carved object of the Kotagedhe Mataram Mosque Gate which is shaped like a diamond.

Source. Research elaboration and author's documentation.

Tables 4 and 5 are descriptions of the alleged learning trajectories that might be found in mathematics content based on the cultural contexts found. This process is a form of ethnomodeling where this stage has entered dialectical analysis as a combination of the ethical and emic approaches previously carried out. The final process is identifying the cultural values of the selected context ornaments. According to the critical person, the cultural value that can be learned from the mosque gate building, which resembles a temple, is tolerance. Even since ancient times, the importance of religious tolerance has been upheld, and the maturity of the people in embracing their respective religions and respecting one another is exemplary. Not only conceptually, the manifestation of the high value of religious tolerance can be seen in cooperation activities. During its construction, the Kotagedhe Mataram Mosque was built in cooperation assisted by Hindus who were around the mosque location. The value of gotong royong, regardless of ethnicity, religion, and race, must also be instilled in students, especially in elementary schools.

Prambanan Temple

The second potential location explored by the research team is Prambanan Temple. To obtain valid information, the researchers conducted interviews and direct observations with two key persons who worked as conservation team members at the Special Region of Yogyakarta Cultural Heritage Preservation Center. The two key persons who became resource persons were Rosita Nur Aharti (as a history expert) and Chandra (as a temple restoration laboratory expert). Interviews and observations were conducted for approximately 2 hours at the Prambanan Temple object.

Prambanan Temple has three layers of courtyards: the outer or outer courtyard, the middle square and the neuron or inner courtyard, with an exterior courtyard. The outer courtyard once consisted of a stone fence; only ruins remain today. The number of small temples outside the central area is around 220, while in the main room, there are 16 temples. The Three Main Temple Buildings. Looking at it from a height, you will see that Prambanan Temple has three main temples.

In addition, in its preparation, Prambanan Temple uses two locking systems, namely the vertical and horizontal interlocking stone techniques. In contrast to the limestone adhesive used for the Kotagehde Mataram Mosque Gate, Prambanan Temple is structured using a locking system so the building can be arranged firmly and not easily collapse. Some examples of lockdowns at Candri Prambanan are as follows:

Figure 6

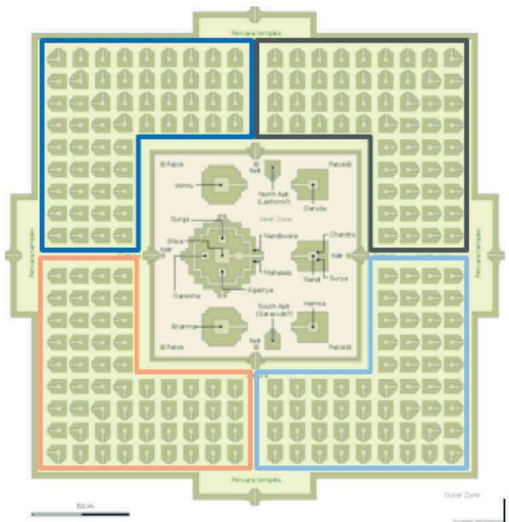
System of Interlocking Stones at Prambanan Temple

Source. Author's documentation.

According to sources, the interlocking stone system used to construct the Prambanan Temple was also carried out at other temples, such as Boko Temple or Borobudur Temple. This system is used by manufacturers, also considering the resilience of buildings in the face of earthquakes. The source said several parts of the temple had been restored in 1928 by the Dutch East Indies (Pre-Independence) Government. This restoration uses cement as an adhesive between stones. However, when the eruption of Mount Merapi in Yogyakarta in 2006 occurred, the temples resulting from the repair were destroyed and collapsed.

Meanwhile, temples that still use the interlocking stone system survive. According to the source, it is precisely because the stones are not connected with cement (into one solid part) that the displacement of the building is more flexible and resistant to earthquakes. Apart from the matter of interlocking stones, an exciting thing that has the potential to have mathematical content is the number and overall size of the temples in the Prambanan Temple complex. The position plan of the Prambanan Temple can be seen in the following Figure.

Figure 7
Design of Prambanan Temple

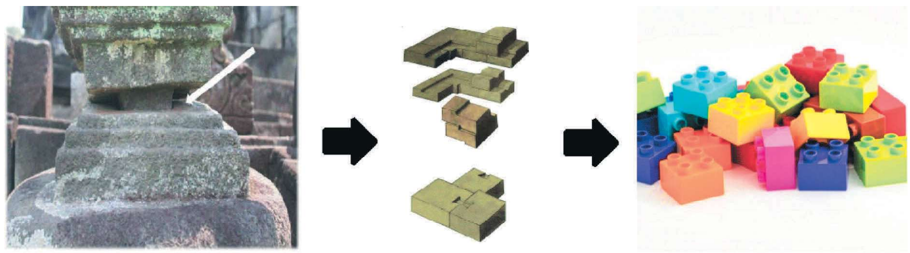


Source. Author's documentation.

The temples outside the main temple complex consist of 4 different types of temples. The difference between the temples lies in the stone carvings and the kala ornaments in each temple. Every kind of temple has a number and arrangement that are systematically arranged with symmetrical spacing. The results of the Interpretation from an ethical point of view of the findings above show some mathematical content that can be elaborated with the context of Prambanan Temple. Researchers found two potential pieces of content teachers can use to learn in schools. The first content is a system of interlocking stones that can be used to practice spatial geometry skills in spatial arrangement. According to J. M. Tenz (1893), the interlocking system is a locking process using stones linked (connected) to each other. A representation of the process of interlocking rocks can be seen in Figure 8 below.

Figure 8

Representation of the System of Interlocking Stones



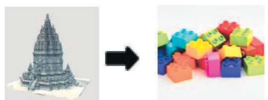
Source. Author's documentation.

Figure 8 shows that the spatial arrangement system uses a stone lock system. This can be idealised with LEGO games where each spatial shape can relate to locks. Research results by Bahjat Altakhayneh (2020); and Keeshan Williams et al. (2012) showed that the constructive process of assembling LEGO to resemble objects could train students' abilities to understand geometric shapes and symmetrical spatial arrangements. This is supported by Van Hiele's theory (Rico-Bautista et al., 2019), which states that the geometry ability is at the deduction level of synthesis, where students can determine the distance between geometric objects and compare concepts between geometric shapes that are already understood. LEGO games can be potential teaching aids for constructing geometric objects. The results of research by Lado Bestra et al. (2022) showed that LEGO games in learning mathematics can develop students' creative thinking skills, collaboration, and concentration in class. In addition, the results of research by Swiya Nath and Dénes Szücs (2014) are also considered to be able to improve the maths skills of elementary school students effectively. In this case, the teacher can also use AR assistance to show the framework of the Prambanan Temple as a reference in constructing the Prambanan Temple using LEGO. The description of the HLT for this learning can be seen in Table 6 below.

Table 6

HLT of Spatial Geometry Capabilities

Hypothetical learning trajectory	Learning activity
Building Surface Area Space Build Volume	Perform measurements of the surface area and volume of the overall geometric shape.
Beam Surface Area Surface Area of a Cube Cone Surface Area	Calculating the number of LEGOs needed to assemble one Prambanan Temple object. Calculating the size of the object that has been arranged at each level (the sides and height of the building).

Hypothetical learning trajectory	Learning activity
	Assembling LEGO bricks to resemble Prambanan Temple objects in groups with teacher assistance.
	Understand the design market for Prambanan Temple stone structures which are visualised with Augmented Reality technology.
	Observing the objects of interlocking stones in Prambanan Temple.

Source. Research elaboration.

Another mathematical content found is the concept of integer operations, especially multiplication operations. Number operations material is a fundamental ability that is very important for elementary school students, especially in the lower grades. Integer operations are an initial ability that will become a bridge for students to learn more complex material at higher rates (Ilukena et al., 2020). The context of the number of temples outside the Prambanan Temple's central courtyard can be used to understand the concept of integer multiplication. According to Monica Wong and David Evans (2007), the multiplication operation can be done utilising repeated addition, so it will be easy to understand the multiplication concept with objects with the same characteristics. Karl W. Kosko's research results (2019) stated that using things students could imagine would facilitate the multiplication thinking process. The learning device that has the potential to be used to teach multiplication material in the context of Prambanan Temple is Augmented Reality media. AR media can visualise the arrangement and number of Prambanan Temples according to previously obtained information. The description of the HLT for this learning can be seen in Table 7 below.

Table 7

HLT Multiplication Concept

Hypothetical learning trajectory	Learning activity
Multiplication of integers is the basic operation of repeated addition of numbers added to themselves a certain number of times	Generalise the concept of multiplication.
Number of Prambanan Temples=Group 1+Group 2+Group 3+Group 4+Group 5 	Repeated summation of Prambanan Temple objects from the smallest to the largest. Grouping each temple according to the shape of the temple.

Hypothetical learning trajectory	Learning activity
	Confirming the number of temples with the help of visualisation from Augmented Reality media.
  	Observing the number and arrangement of buildings at Prambanan Temple.

Source. Research elaboration and author's documentation.

Tables 6 and 7 are descriptions of the suggested learning trajectories that might be found in mathematics content based on the cultural context of the Prambanan Temple. This process is one of the results of ethnomodeling, representing a dialectical approach as a combination of the ethical and emic procedures previously carried out. The final stage is to identify the cultural values contained in the Prambanan Temple object. According to the critical person, three cultural values can be taken from the history of the construction of the Prambanan Temple, namely:

- Religion and Spirituality. Prambanan Temple is a Hindu temple complex dedicated to the gods in Hinduism, namely Brahma, Vishnu, and Shiva. The magnificent and intricate architecture reflects the majesty and power of these gods. The Prambanan Temple also shows the relationship between humans and the spiritual world and the importance of religious practices in daily life;
- Balance and Harmony. The architecture of the Prambanan Temple displays a proportional and harmonious balance between building elements. Each part of the temple has a proper role and function, complementing each other and creating a beautiful visual unity. This balance reflects the concept of harmony in the Hindu cosmos;
- Beauty and Aesthetics. Prambanan Temple is stunning with its intricate and detailed architectural ornaments. The reliefs adorn the temple walls, depicting epic stories and Hindu mythology with fantastic subtlety and beauty. This beauty shows humanity's love for art and aesthetics in creating enchanting places of worship.

Planting System in Yogyakarta

The last location that became the researcher's goal was the Office of the Kedaulatan Rakyat Daily Newspaper. This location was chosen based on the results of an FGD between the researcher and an ethnomathematics research expert who had previously conducted research in Yogyakarta. KR Newspaper has often raised strong cultural themes, one of which is the planting system in Yogyakarta. The researcher managed to meet the key person who has a position as one of the curators at the KR Newspaper, but the source wanted to keep his name private. Therefore, we will use the word "source person" to explain the cultural information conveyed by the critical person Newspaper's key person regarding Yogyakarta's planting system in Yogyakarta has a video fact context that can be used in teaching mathematics at school. In-depth interviews were conducted for approximately 1.5 hours at a predetermined location.

In farming practices, traditional Javanese farmers already know about setting the planting season. This planting season setting is based on the calculation of the Solar year, which in 1 year consists of 365 days. Each year is divided into 12 prey (seasons). The division into 12 prey each year is based on the farmers' knowledge of the appearance of certain stars in the sky. That is why the signs of the constellations in the sky are usually used to determine each prey's start and end times. In addition to determining the start and end of prey, you can also use the length of the human shadow during the day. Because each prey has a different character, one must look for prey sui Table for the plants to be planted in farming practices. The 12 prey can be summarised again into four main prey, namely:

- Katiga's prey includes Kasa, Karo, and Katelu prey, with a length of 88 days;
- Labuh prey includes Kapat, Kalima and Kanem prey, with a length of 95 days;
- Rendheng prey includes prey Kapitu, Kawolu and Kasanga, for 94 days;
- Mareng's prey includes Kasapuh, Desta and Saddha's prey, with a length of 88 days.

Based on information from sources, in Yogyakarta itself until now, some farmers still use the above planting calculations (ethics). Of the five existing regencies, three areas have different planting calculations: Kulon Progo Regency, Sleman Regency, and Gunung Kidul Regency. This difference occurs because there are geographical differences between the three regions. This geographical difference causes differences in the timing of the rains as a sufficient water supply for agriculture in each region. For Kulon Progo Regency, most farmers use Katiga prey with a planting period of 88 days. For the Sleman Regency, farmers often use Labuh prey with a planting period of 95 days. As for Gunung Kidul Regency, farmers mostly use Rendheng prey, which spans 94 days.

Of the three types of cropping calculations in Yogyakarta, the Least Common Multiple (LCM) content can be taught to elementary school students (emic). According to Septi Triyani et al.. (2012), LCM is material studied in elementary school to strengthen the concept of number factors to study more complex material in the future. In addition, according to Sutarto et al. (2021), in arithmetic and number theory, the least common multiple of two numbers is the smallest positive integer divisible evenly by both numbers. Problems in KPK content are often given to students needing clear context. Students are only given numbers and asked to find the value of the KPK using several established methods, such as the Factor Tree and the Biggest Factor Table.

However, according to John Bradford Burkman's research (2013), the ability of KPK that is not accompanied by context will increase the possibility of misconceptions that occur in students. This is because the concept of factors is often difficult for students to understand, especially for students in elementary schools who still cannot think concretely. Furthermore, Edgar John Sintema and José M. Marban (2021) shows that one of the misconceptions that often occurs in students is using the most significant prime number as a divisor. The findings from the context of the planting system in Java can be used as a context in this KPK study. The HLT description can be seen in Table 8 below.

Table 8
HLT of the LCM Concept

Hypothetical learning trajectory	Learning activity
To find the time or point when several events having different periods will occur simultaneously, as in problems about cycles or patterns	Understand the concept of LCM.
88 = 94 = 95 =	Using the LCM concept by means of a factor tree or prime factorisation.
If in January 2023 the three districts planted simultaneously, when did the three districts plant again for the second time?	Determine when the three districts are planted in the same month.
Kulon Progo ➔ 88 days Sleman ➔ 95 days Gunung Kidul ➔ 94 days	Understanding information related to data on planting systems in Yogyakarta from 3 Regencies of Kulon Progo, Sleman, and Gunung Kidul.

Source. Research elaboration.

Table 8 shows the alleged learning trajectory that might occur in learning mathematics for LCM material with planting system content in Yogyakarta. This process is one of the results of ethnomodeling as a manifestation of a dialectical approach that combines ethical and emic approaches. The final stage is to identify the cultural values contained in the object of the Planting Planting System in Yogyakarta. According to the key person, three cultural values can be taken from the planting system in Yogyakarta, namely:

- Harmony with Nature: Traditional cropping systems in Java often emphasise the principles of sustainability and harmony with nature. The timing of planting, the use of local varieties resistant to environmental conditions, and agroecological practices adapted to natural cycles all reflect an understanding and respect for nature as a source of life;
- Connection with Seasons: The traditional practice of planting in Java is often based on seasonal cycles. Planting and harvesting are adapted to changing seasons, reflecting a deep understanding of the changing weather and climate in the area;
- Respect for Cultural Heritage: Traditional planting systems in Java are essential to the cultural heritage passed down from ancestors. These practices have symbolic and ritual values, such as traditional ceremonies associated with planting and harvesting seasons. This helps maintain cultural identity and honours ancestral traditions

Conclusion

The study results show that cultural context can be used in teaching mathematics in elementary schools. The first cultural context found is the Kotagedhe Mataram Mosque which has two mathematical contents. The first content is the mosque's gate as a temple, with Spatial Geometry content in its construction (sociofacts). The second content is the diamond-shaped door of the mosque, which has a folding symmetry content (artifact). The following cultural context is the Prambanan Temple, where interlocking stones are related to the spatial geometric content of three-dimensional structures (ideofacts). Apart from that, there is also multiplication content to calculate the total number of temples in the Prambanan Temple complex (artifacts). The last cultural context is the cropping system, which contains the least common multiple (LCM) in the calculation process (ideofacts).

In addition to context findings, researchers also found potential learning tools teachers can use to use cultural contexts in classroom learning. Learning tools that can be used include folded paper, flat-shaped paper, and LEGO, which can be supported by Augmented Reality technology as a visualisation of the cultural context being studied. For each context and content finding, a Hypothetical Learning Trajectory (HLT) is also prepared, which describes the learning process based on the cultural context and tools that have the potential to improve students' abilities in class.

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